RANGIA COLLEGE

DEPARTMENT OF MATHEMATICS

HOME ASSIGNMENT

6^{th} Semester (General), 2020

Paper : 6.4 (Advanced Calculus **)**

(This Home assignment will be assessed as an Internal Examination (Sessional examination))

The figures in the margin indicate full marks for the questions

1. Answer the following. 1 x 6 = 6

i) Define usual metric on R^2 ($R^2 = R \times R$ is Complex plane).

ii) Is it true that different metric can be defined on the single non-empty set ?

iii) Write the Euclidean metric on R^n.

iv) Define open set in a metric space (X, d).

v) Is the empty set ϕ in a metric space (X, d), closed ?

vi) On the real line (R, d), find whether or not the subset $]0, 4[$ of R is a neighbourhood of 3 ?.

2. Answer the following 2 x 3 = 6

i) On the real line (R, d), show that a singleton set is not open.

ii) In the usual metric space (R, d), show that every open interval is an open set.

iii) In a discrete metric space (X, d), show that every subset is open.

3. Prove that the set R^n of all ordered n-tuples with the function d defined by

$$d(x, y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^{1/2}, \text{ for all } x = (x_1, x_2, \ . \ . \ . \ , x_n) \text{ and } y = (y_1, y_2, \ . \ . \ . \ , y_n) \in R^n$$

is a metric space. 4

4. In a metric space (X, d), prove that the intersection of a finite number of open sets is open. 4

5. In a metric space (X, d), show that a subset F of X is closed if and only if its complement is open. 4