RANGIA COLLEGE
DEPARTMENT OF MATHEMATICS

HOME ASSIGNMENT

6^{th} Semester (General), 2020
Paper : 6.2 (Advanced Calculus **)**

(This Home assignment will be assessed as an Internal Examination (Sessional examination))

The figures in the margin indicate full marks for the questions

1. Answer the following. 1 x 6 = 6

   i) Define usual metric on R^2 ($R^2 = R$ x R is Complex plane).
   ii) Is it true that different metric can be defined on the single non-empty set ?
   iii) Write the Euclidean metric on R^n.
   iv) Define open set in a metric space (X, d).
   v) Is the empty set ϕ in a metric space (X, d), closed ?
   vi) On the real line (R, d), find whether or not the subset]0, 4[of R is a neighbourhood of 3 ?.

2. Answer the following 2 x 3 = 6

   i) On the real line (R, d), show that a singleton set is not open.
   ii) In the usual metric space (R, d), show that every open interval is an open set.
   iii) In a discrete metric space (X, d), show that every subset is open.

3. Prove that the set R^n of all ordered n-tuples with the function d defined by

   $d(x, y) = \left(\sum_{i=1}^{n}(x_i - y_i)^2\right)^{1/2}$, for all $x = (x_1, x_2, \ . \ . \ . \ , x_n)$ and $y = (y_1, y_2, \ . \ . \ . \ , y_n) \in R^n$

   is a metric space. 4

4. In a metric space (X, d), prove that the intersection of a finite number of open sets is open. 4

5. In a metric space (X, d), show that a subset F of X is closed if and only if its complement is open. 4